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TESTS OF THREE LARGE-SIZED REINFORCED-CONCRETE SLABS UNDER CONCENTRATED LOADING

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INTRODUCTION

Numerous instances occur in reinforced-concrete design in which the use of slabs supported at two ends only is required, and in many such cases the critical loading is concentrated at one or more points. Such a condition may exist on slab-bridge floors, box culverts, on floors of buildings where heavy machinery is housed, and in other constructions where loads are concentrated.

If a slab, supported at two ends and carrying a single concentrated load, is imagined to be divided into narrow strips extending from support to support, it would seem reasonable to assume that the strip immediately under the load carries a very large part of it and that the adjacent strips receive a smaller amount, depending upon their distances from the load. The most remote strips, those at the edges of the slab, would then probably receive very little load. The question which concerns the designer of such a slab is that of the relative magnitude of the stresses at different distances from the load.

Up to a few years ago the technical literature on this subject was practically nonexistent, and the result was that engineers relied largely on their judgment when called upon to design slabs subjected to concentrated loads. Very naturally, large variations in load-distribution assumptions were made, and as a consequence there were great differences in the design even when the span and load to be carried were practically identical.

The necessity for definite knowledge on this subject was very forcibly brought to the attention of the engineers of the Office of Public Roads and Rural Engineering a few years ago, and a set of tests was made by one of the authors on slabs of 3-foot and 6-foot span length.¹ These tests gave some useful and rather surprising results that have since been

¹ Goldbeck, A. T. Tests of reinforced-concrete slabs under concentrated loading. In *Amer. Soc. Testing Materials, Proc. 16th Ann. Meeting 1913*, v. 13, p. 858-873, 10 fig. 1913. Discussion, p. 874-883, 4 fig.

verified; and in order to carry the investigation farther, with slabs of longer span than those previously investigated, the present series of tests was undertaken at the Arlington Experimental Farm of the United States Department of Agriculture.

OBJECT OF INVESTIGATIONS

The theory applied to the design of narrow rectangular reinforced-concrete beams involves the assumption that the stress is constant throughout the width of the beam. In a wide slab the stress distribution varies from a maximum at the point of application of the load to a minimum at the extreme edges. Obviously then, if the rectangular-beam theory were applied to the design of slabs under concentrated loads, the width b used in the design formulas can not be taken as the entire width of the slab. The rectangular-beam theory, however, could be utilized in wide-slab design if it were known what width b should be substituted in the design formulas, and it is the object of this paper to explain tests for determining this width and to demonstrate the application of the theory of narrow rectangular beams to the design of wide slabs supported at two ends and subjected to concentrated loads.

EFFECTIVE WIDTH

The width of the slab that should be used in the rectangular-beam formulas when applied to slab design will be termed the "effective width" of the slab. It is that width over which, if the stress were constant and equal to the maximum stress under actual conditions, the resisting moment would equal the resisting moment of a slab of the same depth and full width, but having varying stress distribution. If the straight-line theory of stress distribution from neutral axis to upper fibers is assumed to be applicable to slabs, the resisting moment of a given slab is dependent on the total stress in the concrete or steel at the dangerous section. The total stress in the concrete, however, is governed by the stresses in the top fibers, and these stresses are proportional to the unit deformations. If, then, there are two slabs of equal depth, one having uniform distribution of deformations and the other a varying distribution, but with their maximum deformations identical, they will likewise have equal resisting moments if the summations of the deformations over their respective widths are identical.

In figure 1, which represents a slab in position on two supports with a concentrated load P , is illustrated the method of obtaining "effective width." Strain-gauge readings are taken of the fiber deformations perpendicular to the supports, as indicated at *eg*. These concrete deformation values are plotted to scale, as, for instance, at f/h , giving the deformation curve JHF , inclosing the area $AJHFE$. This curve shows the variation of stress from the center to each of the two free edges of the slab, and the area under the curve is a function of the total concrete-resisting

moment of the slab. The area $BDGI$, obtained by dividing the area $AJHFE$ by its maximum ordinate CH , has the same total concrete-resisting moment with the stress uniformly distributed as the whole slab, and its width BD is that which may be effective in furnishing sufficient resistance under these conditions to carry the load. The width BD , obtained in this manner, is the "effective width."

DESCRIPTION OF APPARATUS

LOAD-APPLYING APPARATUS.—The slabs tested were 32 feet wide, with a span length of 16 feet, and in order to accommodate such extraordinarily large test specimens it was necessary to build special apparatus. Two supports 32 feet long were constructed of reinforced concrete, and embedded in each of them at the center were two loop-welded eyes carrying four 24-inch 80-pound **I** beams 6 feet above the level of the supports

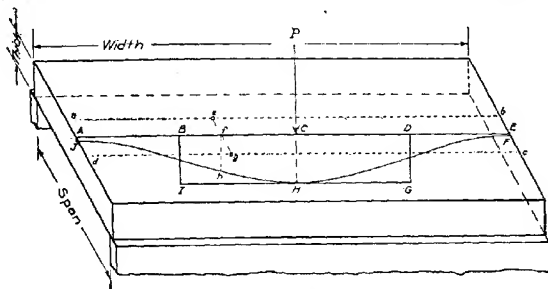


FIG. 1.—Diagram illustrating the method of obtaining "effective width" in reinforced-concrete slab tests

(Pl. XXVI). Loads were applied by means of a hand-operated hydraulic jack mounted between the slab and the overhead **I** beams.

For weighing the loads a specially calibrated chrome-nickel beam (Pl. XXVI) was mounted between the jack and the load-applying **I** beams, and its deflection at the center was a measure of the load applied. This chrome-nickel beam was 7 inches wide, 5 inches deep, and 27 inches in span, and its deflection was measured with an Ames dial reading to 0.0001 inch. The dial was fastened to the beam and its plunger rested on a 1/8-inch square steel rod mounted on the side of the beam at the neutral axis. It was found that by fastening an electric buzzer on this rod more consistent readings could be obtained with the dial. The entire load-applying device was calibrated in a 200,000-pound universal testing machine, and the beam deflections corresponding to known loads were obtained. A deflection of approximately 0.0001 inch occurred for each 500 pounds of load applied. A number of calibrations were made and a calibration curve was plotted. When used for measuring loads, it was only necessary to read the central deflection on the Ames dial and the corresponding load could be read from the curve.

DEFORMATION-MEASURING APPARATUS.—Deformations of the top of the slab were measured at right angles to the supports, and also, in the case of one slab, parallel to the supports, with a Berry strain gauge of 20-inch gauge length. The degree of accuracy attained was probably within 0.0002 inch in that gauge length. Short brass plugs drilled at one end with a No. 55 drill were embedded in the concrete, or in some cases cemented in holes drilled for the purpose; and the movements of these plugs as measured with the strain gauge were considered the fiber deformations.

In the last slab tested (No. 934) deformation readings were also taken of the steel reinforcement, and for this purpose holes were drilled in the steel bars 20 inches apart to accommodate the points of the strain gauge. Although readings were not taken on all of the bars, a sufficient number were measured to determine the distribution of the steel stresses throughout the slab. The layout of strain-gauge points between which readings were made is shown in figures 2, 3, and 4. The arrowheads mark the position of the points on the top of the slab and in the case of slab 934 (fig. 4) the gauge points in the steel are marked by small circles.

DEFLECTION-MEASURING APPARATUS.—The deflection measurements were made in somewhat different ways during these tests, and the apparatus was improved as the tests progressed. In its final form in slab 934, the deflection-measuring equipment consisted of a network of piano wires stretched tightly at a fixed distance above the concrete supports, and being entirely independent of the slab. At the points where measurements were taken, steel plates were set in plaster of Paris on top of the slab. Readings were then made between these plates and the wires by means of a specially designed instrument consisting of a brass stand carrying a bell-crank lever, one end of which touched on the piano wire above and the other end bore on the plunger of an Ames dial. By means of a slow-motion screw the end of the bell-crank lever was adjusted to touch the wire as indicated by an electric buzzer. The dial readings taken at different loads then indicated the deflections at the various points on the slab. This instrument is probably a more convenient form of measuring device than the ordinary inside micrometer and is accurate to 0.004 inch.

DESCRIPTION OF SPECIMENS

All three specimens were 32 feet wide, 16 feet span, and were made of machine-mixed concrete in the proportions 1 to 2 to 4. Potomac River sand and gravel were used as the aggregates, mixed with Portland cement. A rather wet mix was used, and the work of molding was done by laborers at the Arlington Farm who were experienced in work of this character. There was no attempt to make the concrete any better than it would ordinarily be made in the field, but efforts were

directed to secure work thoroughly representative of that obtained under field conditions. The sand was a good grade for use in concrete, and the gravel was clean, well graded, and free from weak pebbles.

The steel reinforcing consisted of $\frac{3}{4}$ -inch plain square bars in slabs 835 and 930, and the bars in slab 934 were $\frac{1}{2}$ -inch square. The yield point of this material is about 39,000 pounds, and the ultimate strength 60,000 pounds per square inch.

The slabs were necessarily built in place on their supports, and the forms were struck at the end of about two weeks. The concrete was sprinkled daily for several weeks during the earlier stages of hardening and was allowed to cure protected from the weather until the destruction of the slab.

Table I contains the essential data concerning the slabs tested.

TABLE I.—Description of reinforced-concrete slabs used in tests¹

Serial No.	Thickness.		Reinforcing.			Modulus of elasticity of concrete.	Central breaking load of slab.
	Total.	Effective.	Size.	Spacing.	Per cent.		
835.....	<i>Inches.</i> 12	<i>Inches.</i> 10½	<i>Inches.</i> ¾ (plain square).	<i>Inches.</i> 10.5	0.75	2,900,000	<i>Pounds.</i> 119,000
930.....	10	8½	¾ (plain square).	8.87	.75	4,000,000	80,000
934.....	7	6	½ (plain square).	5.56	.75	3,000,000	40,000

¹ The slabs were not reinforced transversely.

At the time the slab specimens were made, 8 by 16 inch concrete cylinders were molded from the same mixture and were allowed to cure under the same conditions as the slabs. These were tested later for their crushing strength and modulus of elasticity.

METHOD OF TESTING SLABS

At the age of 28 days the initial strain-gauge and deflection readings were taken with no load on the slab. The first load was then applied through an 8-inch cylindrical bearing block set in plaster of Paris at the center of the slab. Strain-gauge and deflection observations were made again over the entire slab. Due account was taken of the air and concrete temperatures in order to make corrections for any appreciable change occurring during the progress of the tests. The increments of load applied to the different specimens were varied in the different slabs, depending on their thickness, and the aim was to stress neither the steel nor the concrete beyond working limits, also to obtain about five increments of load within the working load.

After readings over the entire slab had been taken, check readings were made at various points; and invariably it was found that these check readings showed an increased deformation in the concrete even though its temperature remained constant. Moreover, upon releasing the load entirely it was found that considerable permanent deformation remained in the concrete. This phenomenon can be attributed only to the "flow" or gradual change in length of the concrete even when under small stresses and is significant, for it shows the importance of the time effect on the relation of stresses and strains in concrete. If the strain readings on the top of the slab, loaded for five or six hours, be used to estimate the stresses in the concrete, based on the initial modulus of elasticity of the concrete, this estimated stress will be greatly in excess of the true stress conditions.

In view of the fact that the deformations which take place in the concrete under a sustained load are continually increasing and remain partially permanent, and that the only deformations of value are those indicative of the stress, all of the final calculations and deductions are based upon results obtained by taking zero deformation readings just before applying the load. Deformations thus obtained by taking the difference between the strain-gauge readings at the zero load and the testing load (all within an hour or so), represent more accurately the elastic deformations and are a better indication of the stress existing in the concrete than those obtained from any initial or previous zero readings.

GRAPHICAL REPRESENTATION OF DATA AND RESULTS

A great amount of numerical data has been taken during the tests of these three concrete slabs. Some of these data were preliminary and served only to indicate methods and limits. Those data which have a direct bearing upon the problem are shown graphically in the accompanying curves (fig. 2-28).

FIGURES 2, 3, AND 4.—The layout of the points in the concrete and the steel over which the strain-gauge readings were taken are shown in figures 2, 3, and 4. In a few cases readings were made between all points, but in general only the readings along a center line (5-6) parallel to the supports were taken, as this gives sufficient data for determining the effective width. In all mention of strain-gauge or deformation readings it should be understood that they are measured between points on a line perpendicular to the supports, unless expressly stated to be otherwise.

FIGURE 5.—Figure 5 shows the variation of the concrete deformations for different concentrated center loads, along the center line of the slab. The ordinates of these curves are influenced slightly by the time factor or "flow" in the concrete; hence, the values for the effective width b are somewhat erratic in their relation to the load.

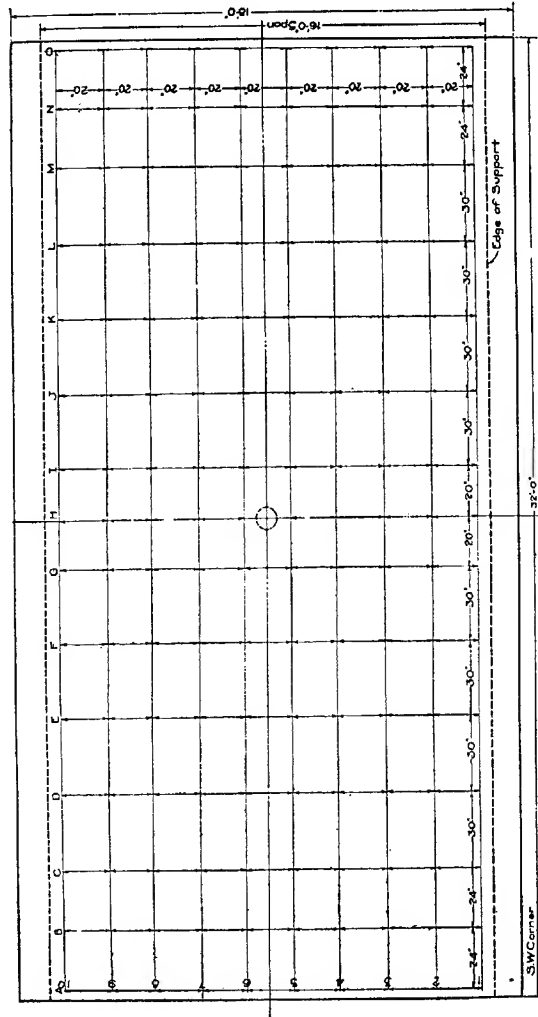


FIG. 3.—Diagram showing location of strain-gauge points on top of slab 1904.

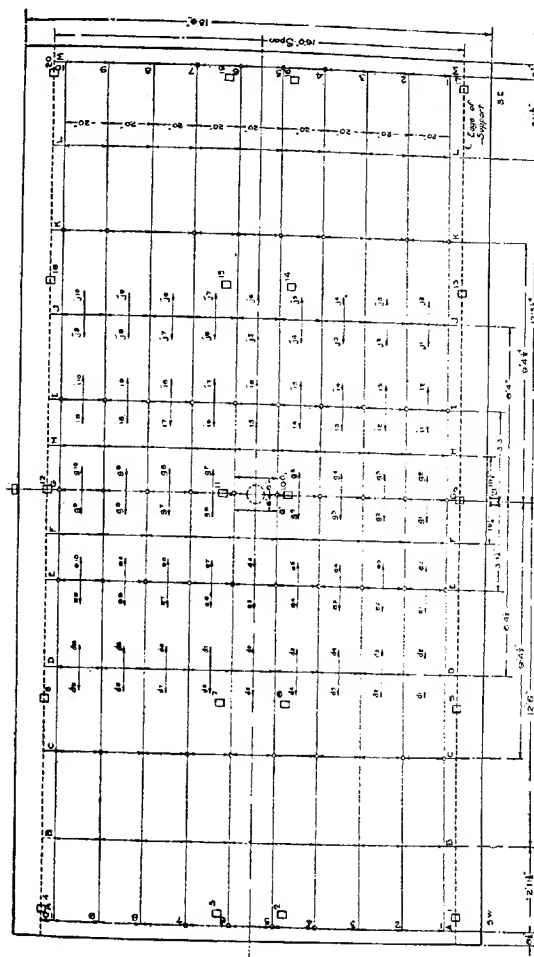


FIG. 4.—Diagram showing location of strain-gauge points on top and bottom of slab 84.

FIGURE 6.—Two curves, A and B, are shown here to indicate the deformations which resulted from the removal of the forms. The flow, or increase in the deformations, is about 80 per cent in three days. The curves C-D, E-F, G-H, I-J, and K-L show the large difference in the deformation and effective width values between those obtained by the use of a zero strain-gauge reading taken several weeks before, with several intervening loadings, and those obtained from a zero reading taken just before the loading. The data and results of curves C, E, G, I, and K are the only ones of value.

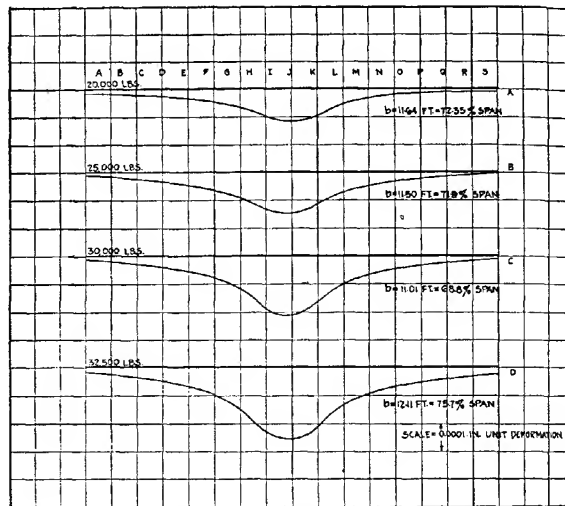


FIG. 5.—Concrete deformation curves for concentrated center load on slab 835.

FIGURE 7.—The difference between these curves shows the magnitude of the set, or permanent deformation, which may occur between two applications of the load, each loading having been applied immediately after a zero reading of the strain-gauge points, with 24 hours intervening between the loadings. The second application shows a smaller deformation than the first. This is true for both the concrete and the steel deformations. The effective widths are based upon the first application of the load.

FIGURE 8.—These curves are shown to emphasize the importance of considering the time factor and its effect upon the deformations in concrete structures. Curve 1 shows the immediate effect of the load. After about 5 hours the load was removed, then again applied 20 hours later

and allowed to remain on for two days, giving curves 2 and 3. The load was then removed, and curve 4 shows the amount of set about two hours later. This set is somewhat reduced after a few days' rest. The values of the effective widths shown in this figure differ very largely and are also indicative of the fact that the time factor is very important.

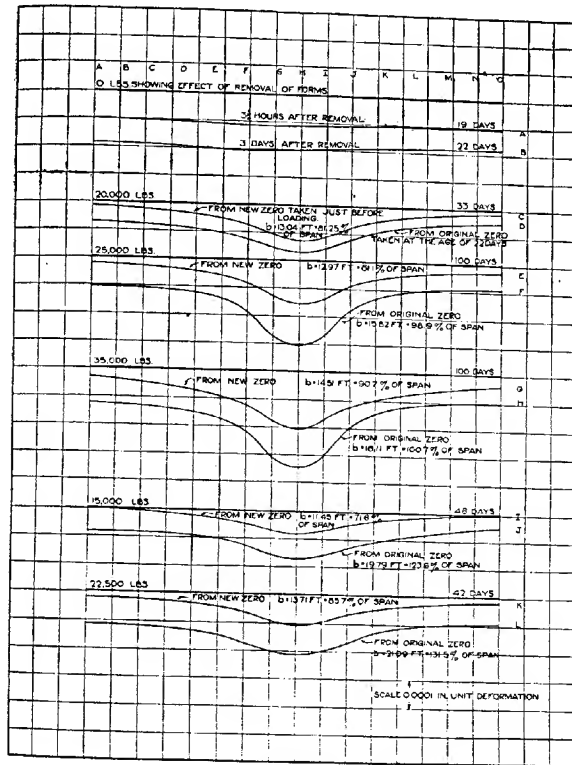


FIG. 6.—Concrete deformation curves for slab 930.

FIGURE 9.—Concrete deformations under 2-point loadings are shown for two-load values. The 40,000-pound load was applied immediately after taking the zero reading, and the deformations taken at once. The load was then increased to the 80,000-pound value and deformations again taken. The whole operation required not over two hours. The local effect at the load points is very pronounced for the larger load.

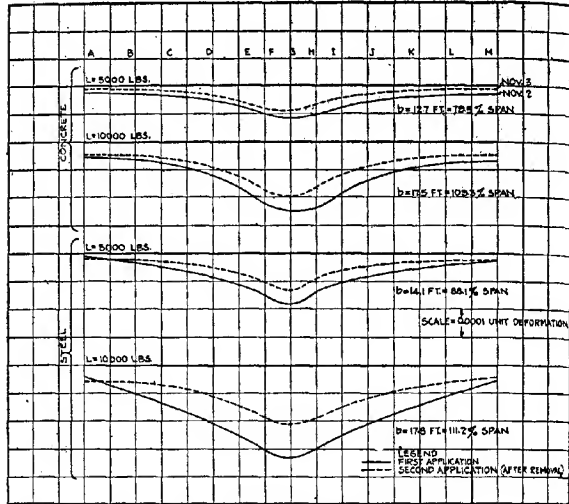


FIG. 7.—Deformation curves for slab 934.

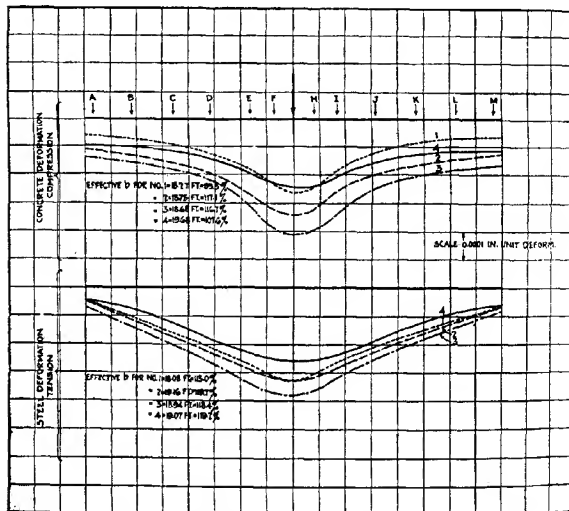


FIG. 8.—Deformation curves for slab 934, computed from first zero reading.

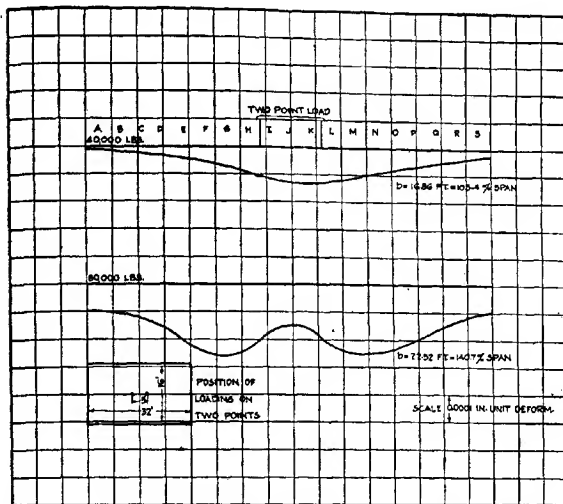


FIG. 9.—Concrete deformation curves for slab 835 with 2-point loading.

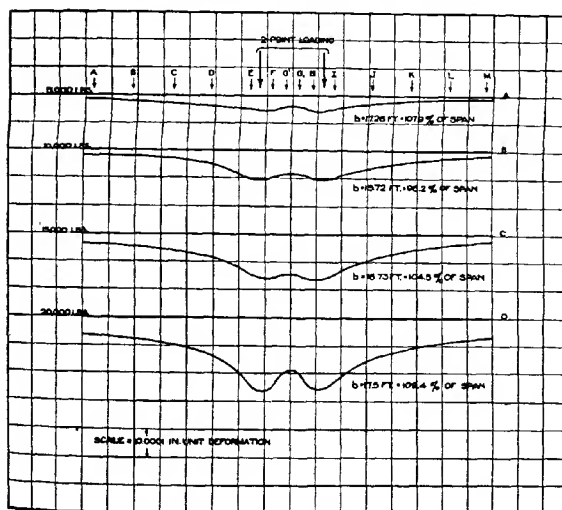


FIG. 10.—Concrete deformation curves for slab 934 with 2-point loading.

The effective width is not materially affected for the 40,000-pound load; but for the 80,000-pound load, which produces the working fiber stress, the effective width is very largely increased.

FIGURES 10 AND 11.—The curves on these figures show a more pronounced local effect in the concrete at the load points than the same character of loading on the thicker slab. It should be noted that for the working load of 20,000 pounds the effective width for this 2-point loading is the same as for the single-point center loading.

FIGURES 12 AND 13.—The results for 4-point loading under different loads are shown in these curves for slabs 835 and 934. The effective

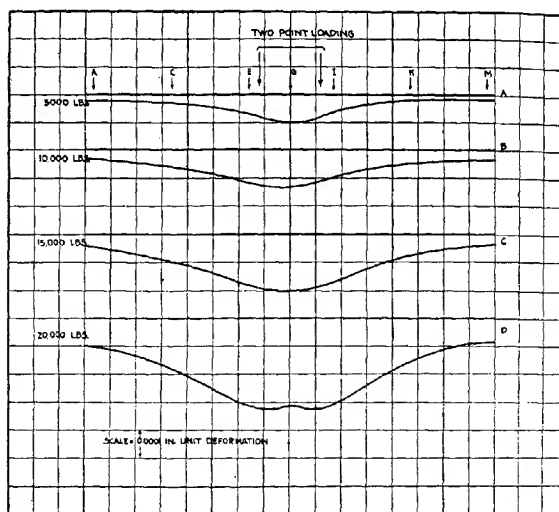


FIG. 22.—Steel deformation curves for slab 934 with 2-point loading.

width is materially affected by the width between the load points; it seems to be increased by not less than 56 per cent of the span length for slab 835, and 93 per cent for slab 934.

FIGURES 14, 15, AND 16.—The deflection data are shown on these figures. The curves are plotted to show the deflection values along a center strip parallel to the supports. In figure 14 curves have been plotted showing the flow and set in the slab under a sustained load and as effected by two applications. Two values for effective widths are shown, which have been obtained from the deflection curves in the same manner as from the concrete deformation curves described above; but these values should not be used in the design of slabs.

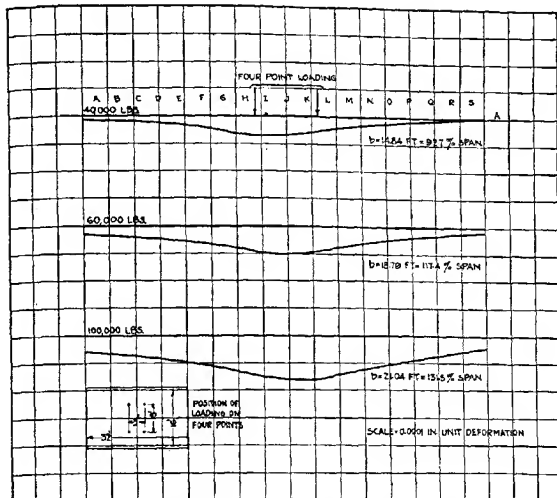


FIG. 12.—Concrete deformation curves for slab 835 with 4-point loading.

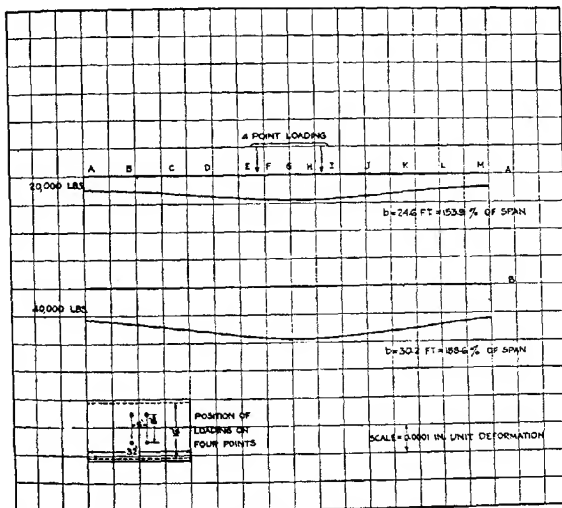


FIG. 13.—Concrete deformation curves for slab 934 with 4-point loading.

FIGURES 17, 18, AND 19.—After each slab was broken the cracks in the top and bottom were drawn to scale. The heavy full lines forming an approximate circle or ellipse around the load point are the tension cracks on the top of the slab caused by the overhang of the ends, after a large center deflection, at about breaking load. The remarkable symmetry of

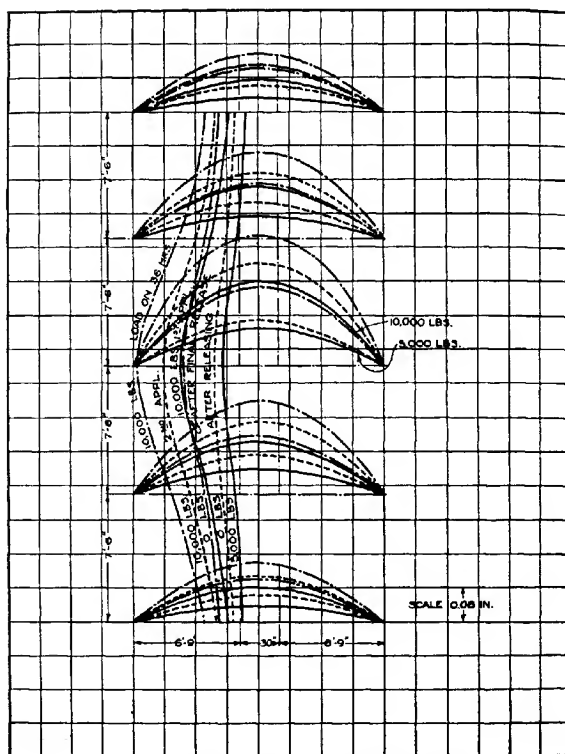


FIG. 14.—Deflection curves for slab 934 on first application of load.

these cracks is worthy of notice. There seems to be no definite relation between the effective width at working loads and the width over which the cracks extended at failure; in fact, it is hardly reasonable that there should be any definite relation, for one case is dealing with safe working stresses within the limit of elasticity, and the other with breaking loads.

Table II shows the breaking loads and their relation to the depth of the slab. Note that the breaking loads are almost directly proportional to the squares of the depths.

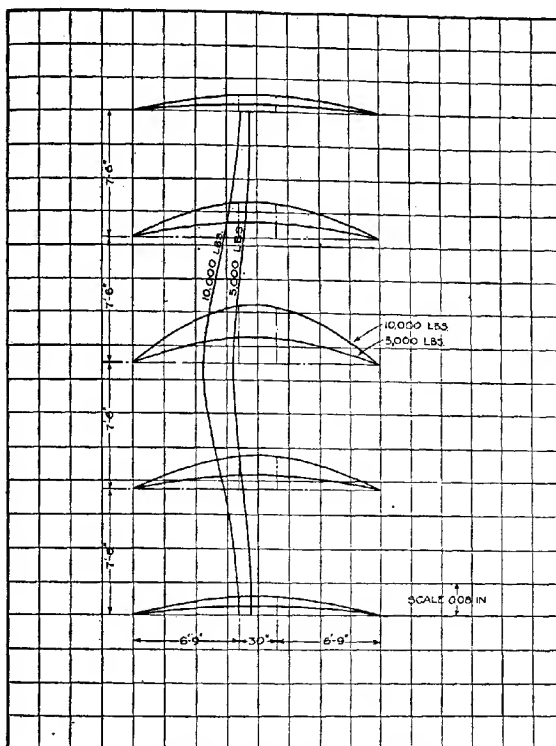


FIG. 15.—Deflection curves for slab 934 on second application of load.

TABLE II.—*Breaking loads of reinforced-concrete slabs and their relation to the depth of slab*

Serial No.	Effective thickness, d .	d^2	Breaking load.	Relations.	
				d^2	Loads.
835.....	10 $\frac{1}{2}$	110.25	110,000	3.06	2.08
930.....	8 $\frac{1}{2}$	72.25	80,000	2.01	2.00
934.....	6	36.00	40,000	1.00	1.00

STRESS DISTRIBUTION OVER THE WHOLE SLAB

For the purpose of determining the distribution of stress over the top of the whole slab, deformation readings at right angles to each other were taken on slab 934 for a working load of 10,000 pounds concentrated at the center.

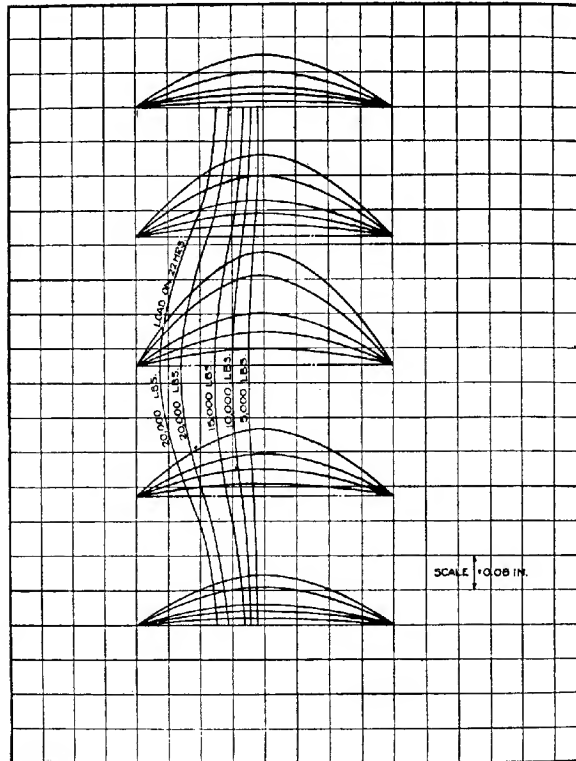


FIG. 15.—Deflection curves for slab 934 with 2-point loading.

FIGURES 20 AND 21.—The deformations measured perpendicular to the supports and plotted on base lines parallel to the supports are shown in figure 20. These curves show the variation of deformations along lines parallel to the supports. The same deformations plotted on base lines perpendicular to the supports, to show the variation in that direction,

are plotted on figure 21. Each curve as shown is an average of the plotted points. The light vertical lines serve only to locate each curve with its base line.

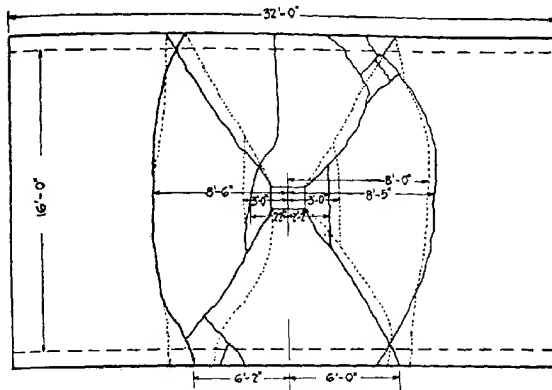


FIG. 17.—Diagram showing effect of breaking load on slab 835.

The variation of the distribution along lines parallel to the supports is somewhat gradual and does not show any sudden changes; but the

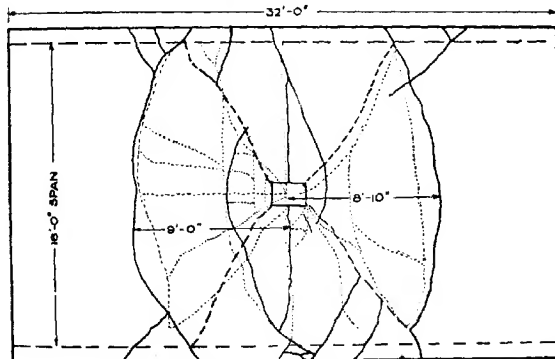


FIG. 18.—Diagram showing effect of breaking load on slab 930.

variation across the span near the center of the slab becomes somewhat critical at and near the load point, and this was more pronounced in the concrete than in the steel. (The steel data are not shown.)

FIGURE 22.—Lateral strain-gauge readings were taken on points parallel to the supports over the middle third of the slab, and these are plotted on base lines both parallel and perpendicular to the supports. The groups of closely drawn parallel lines serve only to connect each curve with its base line. Compression values of the deformations are plotted either to the left or below the base lines, and to the right or above, for values of tension in the concrete. The variations in these lateral deformations are the reverse of those of the longitudinal deformations shown in figures 20 and 21; they are more critical along lines parallel to the supports.

FIGURE 23.—The data of the last three figures have been collected and plotted as "iso-deformation lines," giving a series of lines or contours

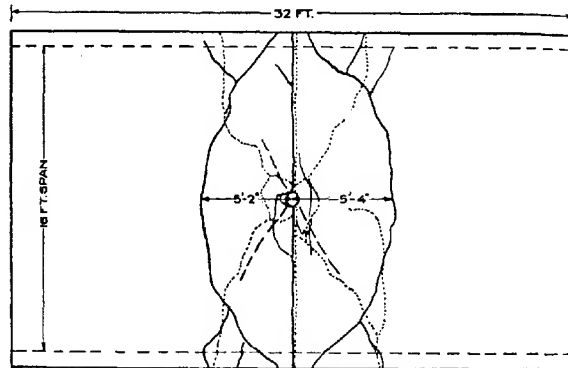


FIG. 19.—Diagram showing effect of breaking load on slab 934.

which represent equal deformations in the concrete on the top of the slab. The lines, as drawn, are averages of the plotted points. Figure 23 (also fig. 26) is more for academic interest and should be of service in the theoretical consideration of stress distribution.

FIGURES 24 AND 25.—These figures are similar to figures 20 and 21, and are plotted in the same manner, except that they represent the distribution of deformations under a working load of 40,000 pounds applied at four points. No lateral deformation readings are shown. The load points are indicated in figure 25. The local effect at the loading points is very pronounced.

FIGURE 26.—The data of the last two figures mentioned have been here collected and show the "iso-deformation lines" for the 4-point loading of 40,000 pounds, total. (See description of figure 23.)

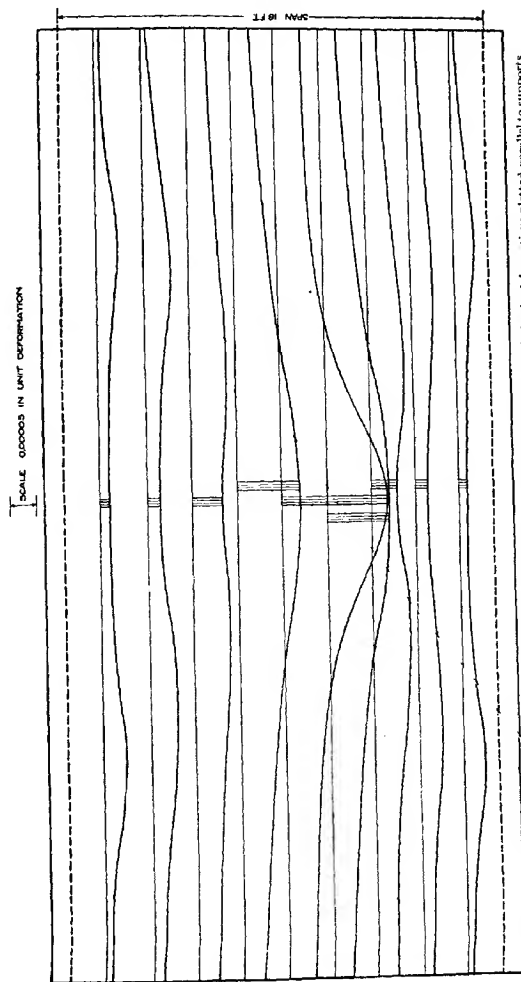
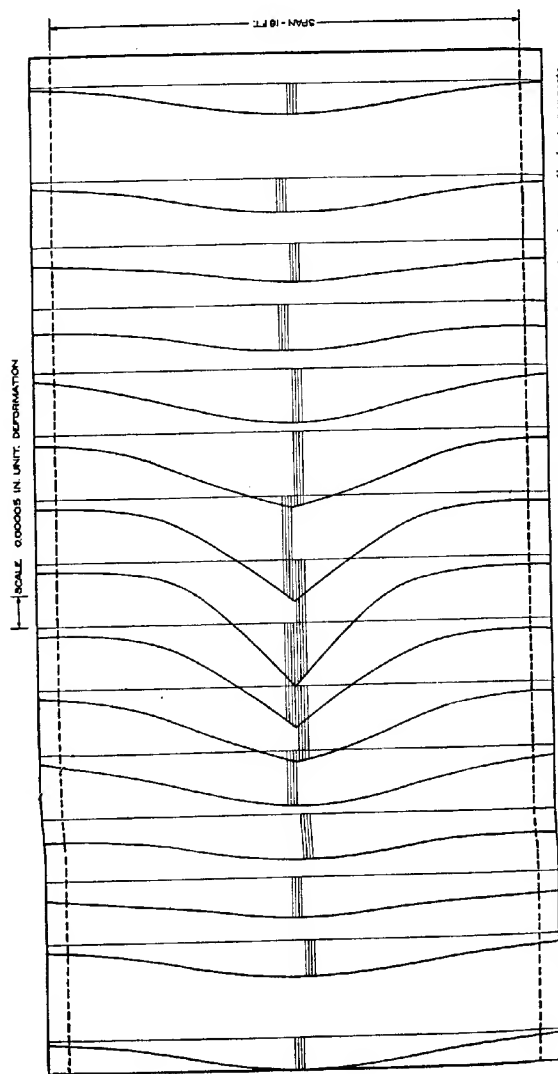


FIG. 30.—Concrete deformation curves for a 10,000-pound concentrated center load on slab 914. Variation in deformations plotted parallel to supports.



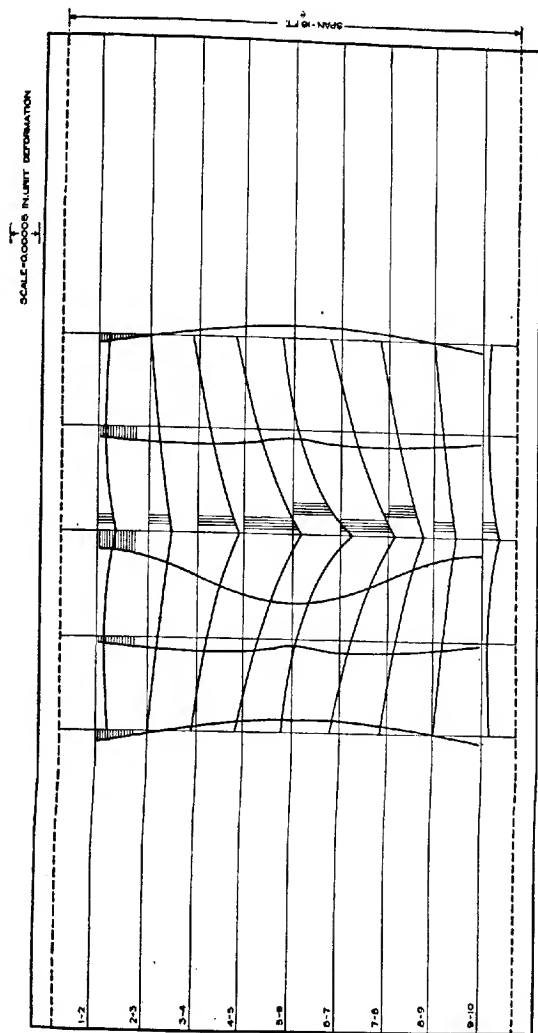


FIG. 21.—Concrete deformation curves for slab 934. Lateral deformations plotted both parallel and perpendicular to supports.

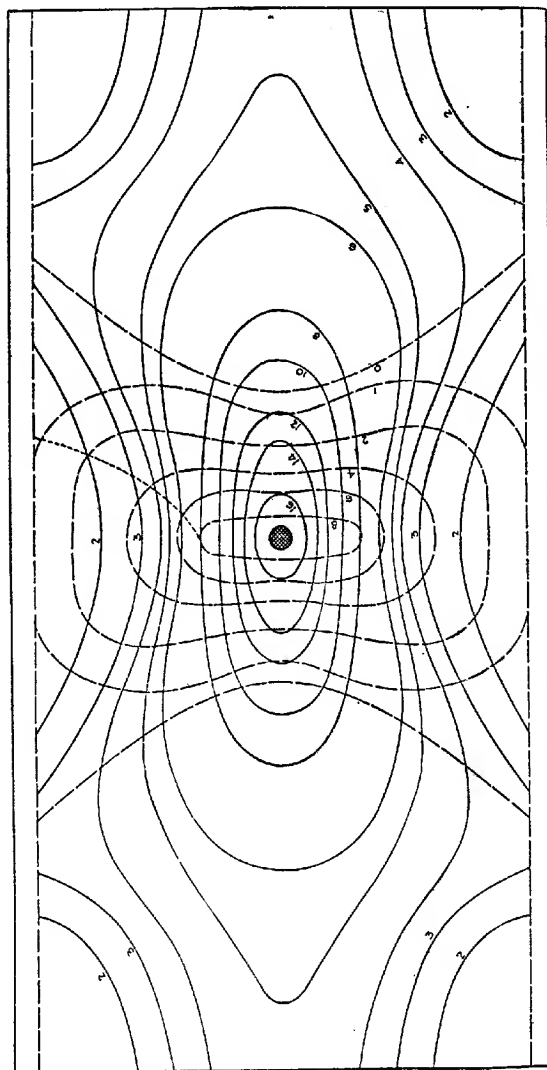


FIG. 23.—Iso-deformation lines for slab 93.5 under concentrated center load.

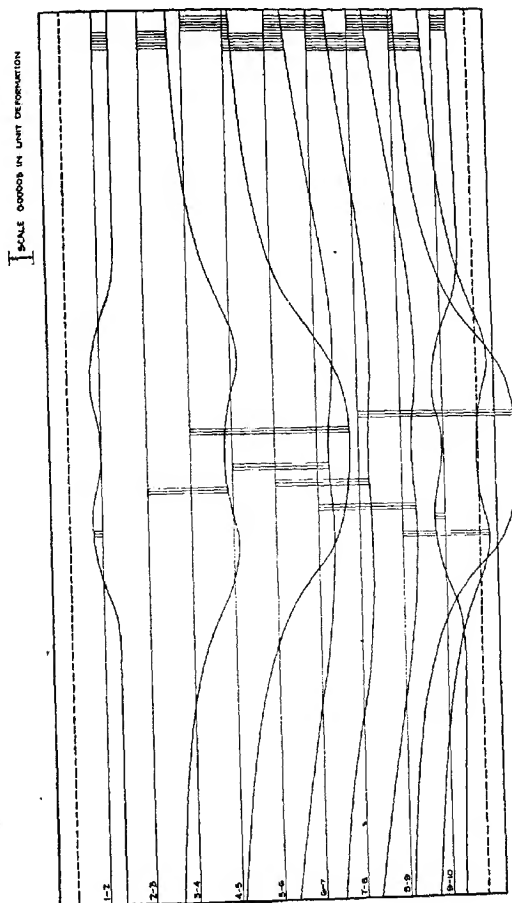


FIG. 34.—Concrete deformation curves for slab 934 under 40,000-pound 4-point loading. Deformations measured perpendicular to supports. Variation of deformations plotted parallel to the supports.

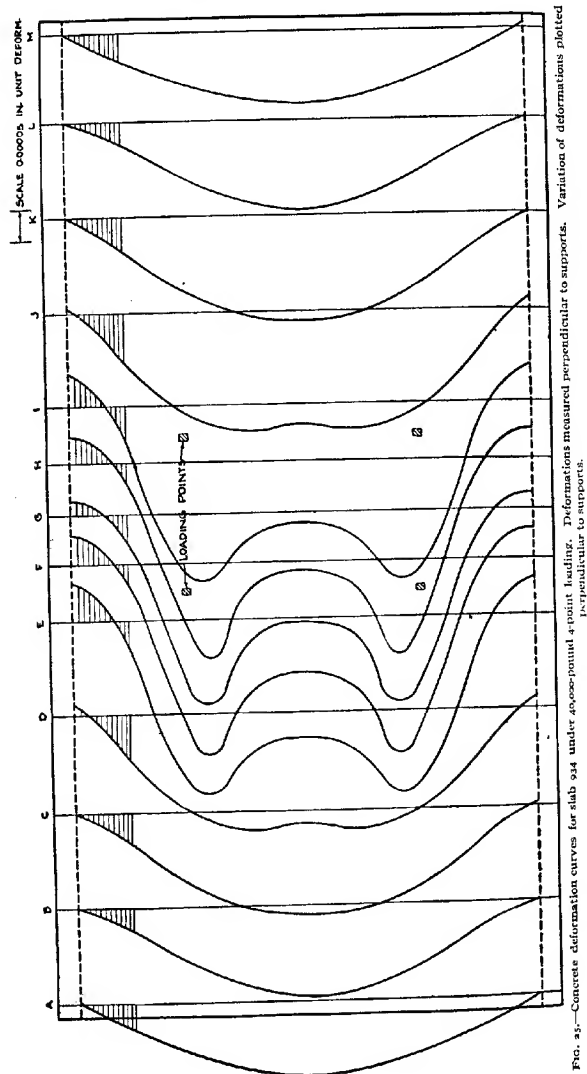


FIG. 23.—Concrete deformation curves for slab 934 under 40000-pound 4-point loading. Deformations measured perpendicular to supports. Variation of deformations plotted perpendicular to supports.

CONCLUSION

If figure 27 is referred to, the influence on the effective width of the magnitude of the load and the manner of interpreting the results may be

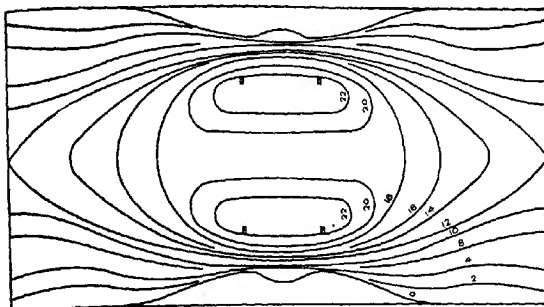


FIG. 26.—Iso-deformation lines for slab 934 under 40,000-pound 4-point loading. Deformations measured perpendicular to supports.

seen. It has been pointed out that the correct method of obtaining deformations is to base all calculations on zero readings taken just before

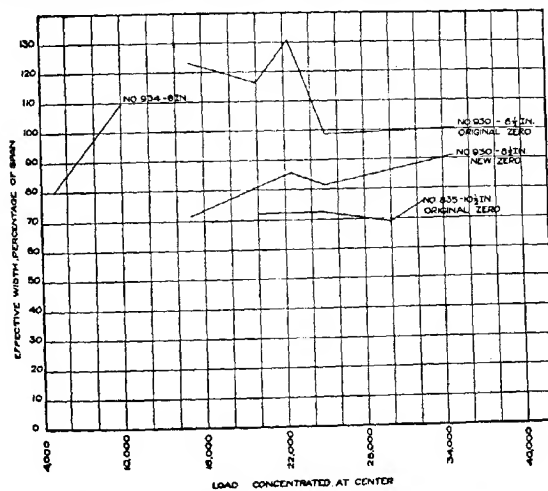


FIG. 27.—Curves showing effective width versus load (concentrated center load).

the load has been applied (designated on the curve as "new zero"). In the case of slab 930, figure 8, note the difference in effective width obtained

depending on the manner of considering the zero readings. The more conservative values are obtained by basing the calculations on the "new zero" readings, as was done in the case of slabs 930 and 934. Note that with an increase in load, the effective width seems to increase slightly. Values for effective width were obtained from the steel deformations, as well as from the concrete deformations, but it was found that the

concrete deformations gave the most conservative widths, and these were therefore plotted.

In figure 28 the effect of variation in thickness of slab on effective width may be seen. Note that as the thickness increases, the effective width decreases, varying from 109 per cent of the span length for a 6-inch slab to 75 per cent of the span for a 10 $\frac{1}{4}$ -inch slab. The least value for effective width shown by these tests is roughly, then, about 0.7 of the

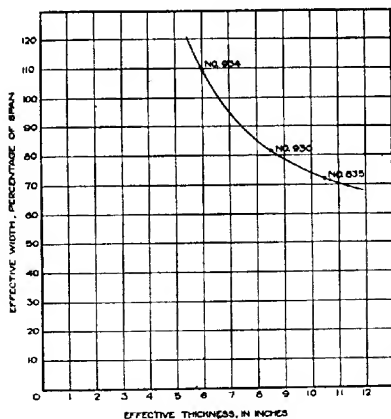


FIG. 28.—Curve showing effective width versus thickness.

span length. Judging from the curve of variation, it would seem that under extremely heavy loads, requiring very thick slabs, the effective width might be decreased as low, possibly, as 0.6 of the span length. However, 0.7 of the span will always be safe, and in general is a sufficiently conservative figure to use.

TABLE III.—Effective widths of reinforced-concrete slabs, 16-foot span by 32 feet wide, for center loading

Center load.	Slab 835 (10 $\frac{1}{4}$ inches effective thickness).	Slab 930 (8 $\frac{3}{4}$ inches effective thickness).	Slab 934 (6 inches effective thickness).
<i>Pounds.</i>			
15,000		11.4 feet=71.6 per cent of span.	12.7 feet=79.5 per cent of span.
20,000	11.6 feet=72.3 per cent of span.	13.0 feet=81.2 per cent of span.	17.5 feet=109.3 per cent of span.
25,000	11.5 feet=71.9 per cent of span.	12.0 feet=81.1 per cent of span.	
32,500	12.1 feet=75.7 per cent of span.		
35,000		14.5 feet=90.7 per cent of span.	
Safe load.	12.1 feet=75.7 per cent of span.	12.0 feet=81.1 per cent of span.	17.5 feet=109.3 per cent of span.

APPLICATION OF RECTANGULAR-BEAM THEORY TO DESIGN OF SLABS UNDER CONCENTRATED LOADS

The usual rectangular-beam design formulas may be applied to the design of slabs by merely substituting for b its value as determined by these investigations, $b = 0.7L$. The corresponding formulas then become—

FOR RECTANGULAR BEAMS

$$(1) M_o = \frac{1}{2} f_c k j b d^2$$

$$(2) M_s = f f_s j b d^2$$

$$(3) p = \frac{a_s}{b d}$$

$$(4) p = \frac{f_s}{f_c} \left(\frac{f_s}{n f_c} + 1 \right)^{1/2}$$

$$(5) k = \sqrt{2 p n + (p n)^2} - p n$$

FOR SLABS UNDER CENTRAL CONCENTRATED LOADS

$$M_o = \frac{1}{2} f_c k j_{10}^2 L d^2$$

$$M_s = f f_s j_{10}^2 L d^2$$

$$p = \frac{10 a_s}{7 L d}$$

$$p = \frac{f_s}{f_c} \left(\frac{f_s}{n f_c} + 1 \right)^{1/2}$$

$$k = \sqrt{2 p n + (p n)^2} - p n$$

It is interesting to note that in substituting for M_o and M_s in formulas 1 and 2 their value $\frac{PL}{4}$, the L 's cancel, showing that the safe load-carrying capacity of the slab is independent of the span; thus—

$$1 \text{ becomes } \frac{PL}{4} = \frac{1}{2} f_c k j_{10}^2 L d^2 \quad \text{or} \quad P = \frac{7}{5} f_c k j d^2$$

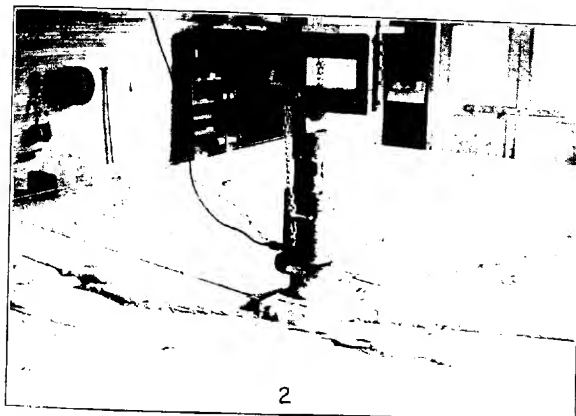
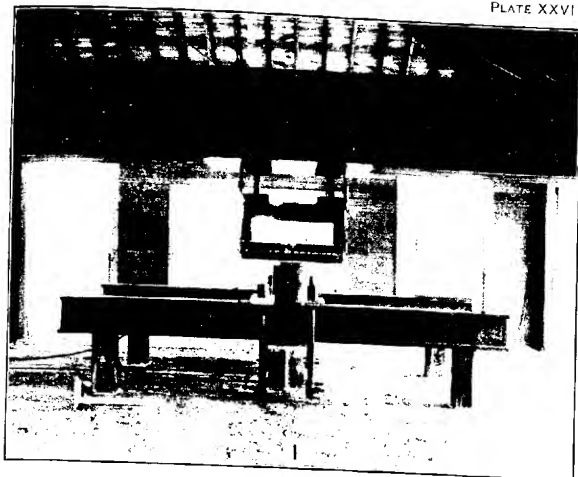
$$2 \text{ becomes } \frac{PL}{4} = f f_s j_{10}^2 L d^2 \quad \text{or} \quad P = p \frac{7}{5} f_s j d^2$$

The above investigations were made on slabs the width of which was twice the span length, so that the stress at the extreme edges was very small. The conclusions must therefore be applied to such cases only. When the ratio of width of slab to span length is less than 2, these conclusions may or may not apply, and additional investigations are now being made to determine the proper value of effective width to use under such conditions.

PLATE XXVI

Fig. 1.—Load-applying and load-measuring apparatus for testing reinforced-concrete slabs, showing set-up for 4-point loading.

Fig. 2.—Load-measuring apparatus and hydraulic jack for testing reinforced-concrete slabs.



OCCURRENCE OF STERILE SPIKELETS IN WHEAT

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INTRODUCTION

The average spike of wheat (*Triticum* spp.) contains from 15 to 20 spikelets, each of which under favorable conditions is capable of producing two or more kernels. Ordinarily, however, the lower two or three spikelets on the spike do not develop. The only indications of their absence are the joints or nodes of the rachis which are thus exposed (Pl. XXVII). Hunt states that often in the cultivated varieties and always in the wild species the lower one to four are sterile. In this paper the term "sterile spikelet" is used to designate those spikelets at the base of the spike which for some reason fail to develop and produce seed. No account was taken of the sterile florets which might occasionally occur within the spikelet. The absent spikelets, as shown by the naked rachis, were the only ones estimated as sterile.

MATERIAL AND METHODS

During the summer of 1915 the writer had the opportunity of making a detailed study of the occurrence of sterile spikelets in a large number of varieties of wheat under test by the Department of Agronomy at the Delaware Agricultural Experiment Station. These varieties and strains of wheat, 188 in number, had been sown the previous autumn by two methods: First, by a grain drill as under ordinary field conditions, at the rate of 7 pecks per acre; second, by the centgener or hill method, leaving the individual plants 6 inches apart each way. By the former method the plants were very close in the rows, which were 8 inches apart. This gave an opportunity to determine to what degree the closeness of the plants or rate of seeding influenced the frequency of sterile spikelets.

The data for each variety were secured in the following manner: The total number of fertile and sterile spikelets were counted on 25 representative spikes of each variety. The means of the fertile spikelets and the sterile spikelets were taken separately and the percentage of sterile spikelets was determined for each variety of wheat. Where the varieties were planted in hills 6 inches apart each way, five plants of five culms each constituted the 25 spikes, the spikelets of which were counted. In this manner the actual number of sterile spikelets and the percentage of the total number of spikelets were determined for the 188 varieties and strains under the two methods of planting.

EFFECT OF RATE OF SEEDING ON STERILITY OF SPIKELETS

It was found (see Table I) that the actual number of sterile spikelets per spike (average of 25 spikes) ranged from 1.84, the lowest, to 5.32, the highest, for varieties in drills; and in hills, from 0.28 sterile spikelets, the lowest, to 3.76, the highest. The percentage of sterile spikelets per average spike in drill rows ranged from 11.5 per cent, the lowest, to 36 per cent, the highest. In hills the percentage of sterile spikelets among the varieties ranged from 1.5 per cent to 23.5 per cent. The mean number of sterile spikelets for all varieties in drill rows was 3.47; in hills, 1.73. The mean percentage of sterile spikelets for all varieties in drills was 21.8 per cent; in hills, 10 per cent.

The data indicate that the spacing of the wheat plants has a direct bearing on the number of sterile spikelets. Wheat planted in hills has more space in which to develop and invariably sends up a greater number of tillers than wheat sown in drills. It has also been observed that the period of maturation is prolonged where the wheat plant has more space. Under these conditions the vegetative activity of the plant is more pronounced, as shown by an increased number of culms, broader leaves, and heavier straw.

TABLE I.—Number and percentage of sterile spikelets on 25 spikes of bearded and smooth varieties of wheat in 1915

Variety.	Bearded or smooth.	Total number of spikelets.		Number of sterile spikelets.		Percentage of sterile spikelets.	
		Drill.	Hill.	Drill.	Hill.	Drill.	Hill.
Acme.....	B	15.36	15.80	3.56	3.72	23.60	23.54
Acme Bred (Maryland).....	B	14.92	15.08	2.96	2.84	19.84	18.83
Acme Improved (Maryland).....	B	13.12	15.68	3.04	2.80	23.17	17.83
Ahrens (Indiana).....	S	16.92	17.92	2.36	1.56	13.94	8.70
American Banner.....	S	16.04	16.92	2.76	2.20	17.20	13.53
American Bronze.....	S	17.88	19.96	3.72	2.88	20.80	14.48
Babcock (Michigan 07664).....	B	14.64	14.72	3.60	2.96	24.50	20.10
Bearded Purple Straw.....	B	15.48	15.04	3.44	2.12	22.22	14.09
Bearded Winter (Michigan 9850).....	B	16.28	16.28	3.76	2.56	23.09	15.72
Bearded Winter Pife.....	B	18.28	17.48	3.80	2.48	20.79	14.18
Beechwood Hybrid.....	S	15.56	16.16	2.48	1.68	16.00	10.39
Beloglina.....	B	15.78	13.52	2.84	1.88	17.90	13.92
Berkeley.....	B	16.12	16.20	3.76	2.12	23.32	13.09
Berkeley Awnless.....	S	14.12	17.08	2.00	2.32	14.10	13.58
Blue Stem.....	S	15.44	15.44	2.84	1.24	18.38	8.03
Broughton.....	S	15.20	15.28	2.68	1.64	17.63	10.73
Buda Dawson (Michigan 310717).....	B	14.36	14.80	3.72	1.80	25.90	12.16
Buda Pest.....	B	15.48	14.48	3.04	1.48	19.63	10.22
Canadian Hybrid.....	S	17.40	15.80	4.12	1.20	23.07	7.59
Century.....	S	15.84	16.24	3.00	1.12	18.93	6.89
China.....	S	17.04	15.68	3.48	2.08	20.42	13.25
Clawson Longberry.....	S	17.64	18.24	3.40	2.00	19.27	10.96
Cooks Brookmont.....	B	13.36	15.16	3.24	2.00	24.25	13.19
Councilman.....	S	15.32	15.76	2.64	1.36	17.23	8.26
Craigs Favorite.....	S	15.48	17.44	3.56	1.76	22.99	10.09
Currells Prolific.....	S	15.52	16.32	3.48	1.12	22.39	6.88
Crimean.....	S	14.64	17.28	3.48	2.04	23.77	11.80

TABLE I.—Number and percentage of sterile spikelets on 25 spikes of bearded and smooth varieties of wheat in 1915—Continued

Variety.	Bearded or smooth.	Total number of spikelets.		Number of sterile spikelets.		Percentage of sterile spikelets.	
		Drill.	Hill.	Drill.	Hill.	Drill.	Hill.
Dawsons Golden Chaff.	S	16.36	17.32	3.00	2.56	18.33	14.77
Defiance.	B	13.60	13.76	3.24	1.80	23.82	11.62
Diamond Grit.	B	18.76	18.76	5.28	3.52	28.14	18.76
Dietz.	B	15.28	15.44	3.24	1.68	21.20	10.87
Dietz Longberry.	B	15.04	16.55	2.86	2.16	19.01	13.19
Doub.	B	15.00	13.52	3.12	1.64	20.80	12.26
Dunlap.	B	15.24	15.00	3.02	1.84	25.72	12.26
Early Harvest.	S	15.60	15.08	2.60	.84	16.66	5.57
Early Red Chief.	S	16.24	17.56	2.64	.40	16.25	2.27
Early Red Clawson.	S	16.76	16.92	3.56	1.16	21.24	6.85
Early Windsor.	S	16.50	17.80	3.48	1.28	21.02	7.19
Eclipse.	B	17.24	19.00	3.02	2.40	23.73	12.62
Egyptian Amber.	B	17.64	17.08	4.72	2.80	26.75	11.63
Enterprise.	S	15.36	16.44	3.40	1.44	22.15	8.75
European Century.	B	16.76	18.48	3.32	2.08	19.80	11.25
Farmers Trust.	B	18.08	17.40	4.48	2.24	24.71	12.87
Fish Head.	B	15.80	18.24	3.00	2.32	18.98	12.71
Fulcaster.	B	15.56	15.28	2.06	2.12	19.02	13.87
Four Row Fultz.	S	16.56	17.76	2.40	.48	14.49	2.76
Jersey Fultz.	S	14.84	15.64	2.72	.60	18.32	3.84
Fultz.	S	16.32	17.00	2.36	.56	14.46	3.18
Fultz Mediterranean.	S	16.40	16.92	1.96	.28	11.95	1.59
Genesee Giant.	B	18.04	19.44	4.20	1.28	23.28	6.58
Giant Square Head.	B	17.40	18.36	4.28	.64	24.59	3.48
Goens.	B	15.36	15.00	3.24	1.04	21.00	6.92
Goens Awnless.	S	15.44	15.52	2.48	.70	16.06	4.96
Gill.	S	15.44	15.12	2.52	.36	16.32	2.38
Glacé.	S	19.00	19.80	4.68	2.08	24.62	10.50
Gold Coin.	S	17.24	17.56	3.60	1.44	20.88	8.20
Golden Bronze.	S	16.24	18.52	3.20	2.04	19.70	11.13
Greening (Michigan 126).	S	16.04	17.92	3.52	1.80	21.94	10.04
Cypsy.	B	17.76	18.84	4.16	2.24	23.42	11.88
Hedges Prolific.	S	15.32	16.80	2.80	.44	18.27	2.16
Hercules.	B	15.16	17.08	3.20	1.88	21.10	11.00
Harvest King.	S	15.72	16.20	2.64	.60	16.79	3.70
Hickman.	S	15.45	15.08	2.44	1.48	15.84	9.43
Hungarian (Michigan 913802).	B	14.92	17.06	3.92	1.80	26.27	10.02
Hybrid Sel. 73.	B	18.00	22.08	3.84	2.56	21.33	11.59
Imperial Amber.	B	16.84	19.88	3.24	2.12	19.23	10.66
International 6 (Michigan 61).	B	15.92	18.68	4.04	2.50	25.37	13.70
Jones Early Red Chaff.	S	15.05	18.00	3.20	1.88	20.47	10.44
Jones Longberry.	S	15.80	17.76	3.28	.92	20.75	5.18
Jones Mammoth Amber.	S	18.32	17.88	4.28	.48	23.36	2.68
Jones Paris Prize.	B	18.66	22.68	5.00	2.36	26.37	10.40
Jones Winter Fife.	S	16.88	18.12	3.00	1.48	17.77	8.22
Kansas Mortgage Lifter.	S	19.60	20.20	3.08	1.04	15.71	5.14
K. B. 2.	B	14.64	16.20	2.60	1.44	17.76	8.88
Kharkov.	S	18.44	20.12	3.84	1.68	20.82	8.34
Klondike.	B	13.96	15.76	3.16	1.16	22.63	7.36
Lancaster-Fulcaster.	B	16.84	17.92	3.16	1.32	18.70	7.42
Lancaster Red.	B	14.32	15.24	3.16	1.44	22.06	9.44
Lebanon.	B	15.84	16.56	4.12	1.52	26.01	9.29
Mammoth Red.	B	14.88	15.84	3.52	2.16	23.65	13.63
Martins Amber.	B	15.80	16.84	2.88	2.00	18.22	11.87
Malakoff.	S	18.24	21.00	4.08	2.32	22.36	11.04
Massey.	B	14.24	14.96	2.48	1.64	17.41	10.96
	S	16.88	20.04	2.44	2.24	14.45	11.12

TABLE I.—Number and percentage of sterile spikelets on 25 spikes of bearded and smooth varieties of wheat in 1915—Continued

Variety.	Bearded or smooth.	Total number of spikelets.		Number of sterile spikelets.		Percentage of sterile spikelets.	
		Drill.	Hill.	Drill.	Hill.	Drill.	Hill.
Meally.....	S	17.72	20.00	2.84	2.16	16.02	10.80
Mediterranean.....	B	15.20	16.00	3.68	1.96	24.12	12.25
Michigan Amber.....	S	15.02	17.52	3.00	1.64	18.84	9.36
Millers Pride.....	B	14.20	10.84	2.64	2.20	18.59	13.00
Miracle.....	B	15.72	16.84	4.40	1.80	27.98	10.68
Missing Link.....	B	17.76	19.84	4.64	2.48	26.12	12.50
Morse.....	S	14.48	16.08	2.24	.56	15.47	3.48
New Amber Longberry.....	B	18.80	20.16	4.84	1.40	25.73	6.99
New Soules.....	S	17.16	17.80	3.58	1.24	20.86	6.96
Nigger.....	B	12.80	14.40	2.68	1.56	20.93	10.83
Nixon.....	S	15.06	15.60	3.50	.84	21.92	5.38
Ohio 5507.....	S	17.28	17.92	3.72	1.04	21.52	5.80
Ontario Wonder.....	S	17.80	20.32	4.20	1.76	23.59	8.66
Orange.....	S	15.08	15.96	2.72	1.68	18.23	10.53
Pesterboden.....	B	15.16	15.96	2.80	1.80	18.46	11.90
Perfection.....	S	14.02	16.44	2.92	.60	19.57	3.64
Plymouth Rock.....	S	16.72	18.88	3.52	2.00	21.05	10.59
Poole.....	S	15.64	18.20	2.72	2.28	17.39	12.52
Pride of Genesee.....	B	18.76	21.48	5.32	3.76	28.35	17.50
Prosperity.....	S	16.84	19.56	3.24	1.60	19.25	8.17
Purple Straw.....	B	15.28	20.44	2.72	2.32	17.80	11.35
Red Cross.....	S	16.60	20.32	3.44	1.24	20.72	6.10
Red Hussar.....	B	12.76	18.12	2.88	.72	22.57	3.97
Red Rock.....	B	13.32	15.68	3.36	1.92	25.22	12.24
Red Wave.....	S	18.16	19.52	4.24	1.56	23.34	7.83
Reiti.....	B	17.20	21.80	4.44	2.20	25.81	10.18
Reliable.....	B	15.88	18.68	3.84	2.24	24.18	11.99
Rochester Red.....	S	15.06	18.80	3.96	1.68	24.81	8.93
Rocky Mountain.....	B	15.00	15.68	3.96	1.92	26.40	12.24
Royal Red Clawson.....	S	14.06	16.52	3.24	.48	21.65	2.90
Rudy.....	B	12.52	14.20	2.72	1.04	20.90	7.32
Rudy Hard.....	B	13.84	16.12	2.92	1.32	21.02	8.18
Ruperts Giant.....	S	17.24	19.80	4.40	1.00	25.51	8.03
Rural New Yorker.....	S	16.92	19.56	3.72	1.44	21.98	7.36
Russian Amber.....	B	15.84	19.52	4.80	2.76	30.39	14.13
Shepherds Perfection.....	B	16.64	20.56	4.56	2.80	27.40	13.61
Silver Sheath.....	B	13.52	17.52	3.68	2.40	27.14	13.69
Silver Wave.....	B	17.32	19.48	4.76	2.88	27.48	14.79
Smiths Rustproof.....	S	16.72	20.68	4.28	2.72	25.59	13.15
Soumans Champion.....	B	16.80	19.28	3.76	1.88	22.38	9.74
Spayde.....	B	16.52	19.76	3.16	1.56	25.18	7.89
St. Louis Grand Prize.....	S	18.00	20.40	3.44	1.24	19.11	6.07
Stone.....	B	13.00	16.96	3.84	2.32	29.53	13.68
Swamp.....	B	14.88	17.88	3.68	.84	24.73	4.69
Theiss.....	B	13.28	17.52	3.84	1.60	28.93	9.70
Turkey Red.....	B	14.44	16.80	2.76	1.72	19.11	10.17
Turkish Amber.....	B	15.48	16.88	3.28	1.24	21.18	7.34
Velvet Chaff.....	B	16.80	18.04	4.04	1.88	24.04	10.42
Valley.....	B	16.68	17.76	4.60	2.32	27.57	13.06
Wayside Wonder.....	B	14.32	17.68	3.92	1.68	27.37	9.83
Whedding.....	S	13.80	15.60	2.40	.92	17.39	5.89
White Eldorado.....	S	15.96	18.36	3.48	1.24	21.80	6.75
Wyandotte Red.....	S	14.64	16.96	2.80	1.16	19.12	6.82
Tennessee 3508.....	B	18.12	22.48	5.52	3.12	30.49	13.87
Tennessee 3609.....	B	17.60	19.68	4.40	2.20	25.00	11.17
Tennessee 3611.....	B	15.16	20.16	4.32	2.76	28.49	13.69
Tennessee 3614.....	B	15.28	18.40	3.80	2.40	24.86	13.04

TABLE I.—Number and percentage of sterile spikelets on 25 spikes of bearded and smooth varieties of wheat in 1915—Continued

Variety.	Bearded or smooth.	Total number of spikelets.		Number of sterile spikelets.		Percentage of sterile spikelets.	
		Drill.	Hill.	Drill.	Hill.	Drill.	Hill.
Tennessee 3617.....	B	17. 16	20. 44	4. 68	2. 36	27. 27	11. 54
Tennessee 3277.....	B	18. 36	21. 60	5. 40	2. 64	29. 41	12. 22
U. S. 2980.....	B	14. 84	17. 08	2. 92	2. 00	19. 67	11. 70
U. S. 3608.....	B	17. 02	21. 24	4. 84	2. 72	27. 06	12. 80
U. S. 3609.....	B	17. 12	19. 68	4. 36	2. 20	25. 46	11. 17
U. S. 3610.....	S	18. 16	21. 56	3. 52	1. 68	19. 38	7. 79
U. S. 3612.....	B	17. 52	21. 36	4. 48	2. 52	25. 57	11. 79
U. S. 3613.....	B	15. 00	20. 40	3. 92	2. 56	26. 13	12. 54
U. S. 3614.....	B	16. 40	19. 20	3. 40	1. 80	20. 73	9. 36
Abundance.....	S	16. 44	17. 76	2. 60	2. 00	15. 81	11. 26
Auburn Red.....	B	16. 08	16. 88	3. 40	2. 32	21. 15	13. 74
Australian Red.....	B	14. 52	15. 84	3. 24	2. 12	22. 31	13. 38
Banat.....	B	13. 72	15. 52	3. 48	2. 64	25. 36	17. 01
Bulgarian.....	B	14. 88	16. 68	4. 16	2. 04	27. 95	12. 23
California Red.....	S	14. 96	15. 76	2. 88	. 32	19. 25	2. 03
Davidson.....	S	15. 95	17. 44	1. 84	. 80	11. 52	4. 58
Deitz Amber.....	B	14. 52	15. 88	3. 80	1. 24	26. 17	7. 80
Deitz Mediterranean.....	R	14. 44	15. 64	3. 50	1. 32	24. 25	8. 43
Early Pearl.....	S	13. 04	13. 02	2. 56	. 72	19. 64	5. 17
Early Ripe.....	S	15. 08	16. 08	2. 80	. 68	18. 56	4. 22
Economy.....	S	14. 48	15. 36	2. 72	1. 16	18. 78	7. 55
Egyptian.....	B	14. 08	17. 20	5. 08	1. 80	36. 07	10. 46
Farmers Friend.....	B	13. 00	14. 40	2. 06	1. 68	22. 76	11. 69
Chirka Winter.....	B	15. 76	18. 56	3. 02	2. 12	28. 88	11. 36
Goings.....	B	15. 16	14. 80	3. 00	1. 04	19. 78	7. 02
Grand Prize.....	S	17. 76	19. 72	3. 36	2. 00	18. 91	10. 14
Invincible.....	S	17. 02	20. 60	4. 36	2. 32	24. 33	11. 26
Jones Red Wave.....	S	18. 20	20. 21	4. 24	1. 64	23. 29	8. 15
Kentucky Bluestem.....	S	14. 84	17. 28	3. 12	1. 40	21. 02	8. 10
Lancaster.....	B	14. 36	15. 16	3. 48	1. 96	24. 23	12. 92
Lehigh.....	B	13. 96	17. 16	3. 52	2. 88	25. 21	16. 78
Pedigree Giant.....	B	18. 24	19. 40	3. 80	1. 40	20. 83	7. 21
Red May.....	S	14. 32	16. 84	2. 64	. 36	18. 43	2. 13
Reiti.....	B	14. 72	17. 88	3. 44	1. 68	25. 14	9. 39
Sibleys New Golden.....	B	14. 44	17. 84	3. 56	1. 52	24. 65	8. 54
Texas Red.....	B	13. 04	17. 28	3. 80	1. 76	29. 14	16. 18
Treadwell.....	B	15. 24	18. 00	4. 60	1. 72	30. 18	9. 55
Tuscan Island.....	B	15. 32	16. 72	3. 92	2. 00	25. 58	11. 96
Uta.....	B	13. 32	16. 04	3. 80	1. 24	28. 52	7. 73
Winter Chief.....	S	15. 44	17. 56	2. 40	. 44	15. 54	2. 50
Winter King.....	B	14. 16	15. 04	3. 24	1. 28	22. 88	8. 51
Wisconsin 13.....	B	13. 12	16. 36	3. 76	1. 24	28. 65	7. 58
Leaps Prolific.....	S	17. 06	18. 88	2. 28	. 68	13. 36	3. 60
Average.....		15. 85	17. 13	3. 47	1. 73	21. 88	10. 09

Of the 188 varieties and strains of wheat under observation, 108 were beardless and 80 bearded. To determine whether the presence or absence of awns as a morphological character was in any way correlated with the occurrence of sterile spikelets, the varieties were tabulated so as to show the distribution of bearded and of beardless varieties with reference to the percentage of spikelets (see Table II). The data in this case were taken from the varieties sown in drills.

TABLE II.—Arrangement of bearded and beardless varieties of wheat with reference to the percentage of sterile spikelets

Percentage of barren spikelets.	Total number of varieties.	Number of beardless varieties.	Number of bearded varieties.	Percentage of each class to total number of—	
				Beardless varieties.	Bearded varieties.
11 to 15.....	8	8	0	10.0	0
15 to 17.....	12	12	0	15.0	0
17 to 19.....	27	19	8	23.7	7.4
19 to 21.....	32	18	14	22.5	12.9
21 to 23.....	30	14	16	17.5	14.8
23 to 25.....	31	7	24	8.7	22.2
25 to 27.....	25	2	23	2.5	21.2
27 to 29.....	17	0	17	0	15.7
29 to 31.....	5	0	5	0	5.5
Total.....	188	80	108	100.0	100.0

Table II shows that the bearded varieties as a class have a higher percentage of sterile spikelets than the beardless wheats. There are 20 of the 80 varieties of beardless wheat which have more than 15 per cent of sterile spikelets, while not a single variety of bearded wheat has less than 17 per cent of sterile spikelets. Of the 108 bearded varieties 45 have not less than 25 per cent of sterile spikelets. Only two of the 80 beardless varieties have 25 per cent of sterile spikelets. The average percentage of sterile spikelets for all the beardless varieties is 17.8; for the bearded, 24.1; a difference of 6.1 per cent in favor of the beardless varieties. The individual variety having the lowest percentage, 11.5, was beardless, while the variety having the highest percentage of sterile spikelets, 36.7, was bearded. All of the varieties which are mentioned above were sown under like conditions of soil preparation and fertilization and planted at the same time.

EFFECT OF TIME OF SEEDING ON STERILITY

The next step was to determine the effect of time of seeding and of soil treatment on the frequency of sterile spikelets. As it happened, an experiment was already under way on different dates of sowing wheat, including two varieties, one bearded and the other beardless, on both fertilized and unfertilized soil. These plants were in hills 6 inches apart each way. In the manner followed above, the total number of spikelets and that of sterile spikelets per spike were combined, and the average was determined for the two varieties under different dates of planting on both treated and untreated soil (Table III).

TABLE III.—Effect of date of planting on the number of sterile spikelets in 25 spikes of two varieties of wheat on fertilized and on unfertilized soil

RED WAVE (BEARLESS)						
Date of planting.	Total number of spikelets.		Number of sterile spikelets.		Percentage of sterile spikelets.	
	Fertilizer.	No fertilizer.	Fertilizer.	No fertilizer.	Fertilizer.	No fertilizer.
Sept. 17.....	21.4	17.7	2.8	2.1	13.4	12.1
24.....	20.5	18.4	2.2	1.8	11.1	9.7
Oct. 1.....	20.0	20.3	2.2	2.2	11.1	10.8
8.....	21.5	18.8	2.1	1.5	10.1	8.4
15.....	21.3	19.9	2.2	1.6	10.6	8.4
22.....	19.7	20.9	1.0	1.2	5.4	5.7
Average.....	20.7	19.3	2.1	1.7	10.3	9.3

MIRACLE (BEARDED)						
Sept. 17.....	16.7	15.0	2.3	1.5	13.8	10.4
24.....	16.2	14.9	2.6	1.2	16.4	8.5
Oct. 1.....	18.9	15.5	2.6	1.8	14.1	11.5
8.....	16.0	15.4	3.1	1.8	19.7	12.1
15.....	15.7	16.8	1.6	1.8	10.1	10.9
22.....	16.1	15.4	.9	.4	5.7	2.8
Average.....	16.6	15.5	2.2	1.4	13.3	9.4

Table III shows that the number of sterile spikelets per spike varies considerably from the earliest seeding, September 17, to the latest, October 22, but in no regular manner. The latest seeding in every case shows the smallest number of sterile spikelets. This holds true for both varieties and under both soil conditions. If the average is taken of the number of sterile spikelets under the six different dates of seeding, it is found that there are more sterile spikelets where fertilizer was used than where no application was made. This also holds true for both varieties. Expressed as a percentage, the average of sterile spikelets for the different rates of seeding with the beardless variety is 10.3 per cent where fertilizer was used and 9.3 per cent on untreated soil. That of the bearded variety was 13.3 per cent of sterile spikelets as an average for the different dates of seeding on treated soil and 9.4 per cent on the untreated. It will be noted that the latest seeding of each variety has as many spikelets as the earliest, and that there are more than twice as many sterile spikelets in the latter than in the former. This may be partially accounted for by the fact that the later plantings did not have a full stand of plants, thus giving the individual wheat plant more space. This explanation is in accord with results obtained under the different methods of seeding (see Table I)—that is, that fewer sterile spikelets were found in the thinner plantings.

The tillering in the early plantings was nearly 100 per cent greater than in the later plantings. The tillering for each variety on fertilized soil for a given date was 50 per cent greater than where no fertilizer was used. The general effect of the date of seeding seems to indicate a tendency toward a smaller percentage of sterile spikelets in the later seedings. The relation of the number of sterile spikelets to yield does not seem to affect the yield seriously, since the fertilized wheats produced two or three times as much grain per spike as the unfertilized. The difference in yield per spike seems to be due largely to quality (size) of kernel.

TABLE IV.—*Relation of the effect of different fertilizers and combinations of fertilizers to the occurrence of sterile spikelets*

Treatment.	Dawson's Golden Chaff (smooth).			Lehigh (bearded).		
	Total num- ber of spike- lets. ¹	Num- ber of sterile spike- lets. ¹	Per- cent- age of sterile spike- lets.	Total num- ber of spike- lets. ¹	Num- ber of sterile spike- lets. ¹	Per- cent- age of sterile spike- lets.
Nitrogen, phosphorus, and potassium.....	16.8	1.36	8.0	17.9	1.32	7.3
Nitrogen and phosphorus.....	18.2	1.68	9.2	18.2	2.08	11.4
Phosphorus and potassium.....	18.2	1.92	10.2	17.2	1.80	10.4
Nitrogen and potassium.....	18.2	1.08	5.9	17.3	.92	5.2
None.....	17.0	1.05	6.1	16.7	1.01	6.0
Nitrogen.....	16.0	.92	5.4	18.0	1.36	7.5
Phosphorus.....	15.9	1.56	9.7	15.0	1.40	9.2
Potassium.....	17.0	1.32	7.7	16.8	1.24	7.3

¹ Average of 23 spikes.

EFFECT OF FERTILIZERS ON STERILITY

The effect of different elements of plant food, singly and in combination, on the number of sterile spikelets is seen in Table IV. The wheat was planted by the centgener method, the individual plants being 6 inches apart each way. On each of the plots sufficient fertilizer of each mineral ingredient was supplied to produce a 50-bushel crop of wheat, provided that it were all used. The nitrogen was applied for a 25-bushel crop, it being assumed that the soil carried a fair reserve of this element. The nitrogen was applied in equal parts by weight of nitrate of soda and dried blood; the phosphoric acid was carried as acid phosphate and the potash as muriate of potash. It will be noted that where the fertilizers were applied singly nitrogen gave the lowest percentage—6.4—of sterile spikelets as an average for the two varieties. Potash came next with 7.5 per cent, and phosphoric acid stood highest, with 9.4 per cent of sterile spikelets. Where two elements were used in combination, phosphoric acid and potash led, with an average of

10.4 per cent for the two varieties; phosphoric acid and nitrogen combined gave 10.3 per cent of sterile spikelets, while nitrogen and potash gave 5.4 per cent. Since phosphoric acid gave the highest percentage of sterile spikelets when used alone, it would seem that this element of plant food is largely responsible for the sterile spikelets, as in every combination in which it is used the number of sterile spikelets is greater than where nitrogen and potash are used singly or in combination. The untreated plot gave 6 per cent of sterile spikelets, the lowest for the series except where nitrogen and potash were used in combination, which gave 5.5 per cent. The complete fertilizer gave an average of 7.6 per cent of sterile spikelets. From these data it would seem that there is a tendency for phosphoric acid to produce a larger percentage of sterile spikelets than either potash or nitrogen. However, the fairly high percentage of sterile spikelets in the case of the wheat treated with phosphoric acid did not affect the yield per plant or spike. Under this treatment the yield and quality of the grain surpassed that under either nitrogen or potash.

CORRELATIONS

In order to determine what relation might exist between the total number of spikelets per spike and the number of sterile spikelets, the readings constituting the averages for the 25 spikes of each variety were arranged in correlation tables. The beardless varieties form one table and the bearded the other. Thus, the readings were the average of each variety and the array or distribution in the table was made up of varieties. The data were secured from the plants in hills. Since the number of spikelets per spike in a large measure determines the length of spike, the relation found will be closely associated with the length of the spike. In Table V, which includes the beardless varieties, the coefficient of correlation between the number of sterile spikelets and the total number of spikelets is 0.543 ± 0.054 . The bearded varieties show a correlation which is expressed as $r = 0.598 \pm 0.041$. It appears that the number of sterile spikelets per variety bears a direct positive correlation to the total number of spikelets or the length of head. The varieties with the shorter spikes have decidedly fewer sterile spikelets. The relation between the number of spikelets and the length of spike may not be close, inasmuch as there may be more or less range among varieties as to the condensation or closeness of the spikelets on the spike. However, the long spikes are made up of a relatively larger number of spikelets than the short ones, and the actual percentage of sterile spikelets may be smaller in the long spikes, as will be pointed out later.

TABLE V.—Correlation between the number of sterile spikelets and the total number of spikelets in beardless and bearded varieties of wheat

BEARDESS VARIETIES ¹									
Number of sterile spikelets.	12 to 13.	13 to 14.	14 to 15.	15 to 16.	16 to 17.	17 to 18.	18 to 19.	19 to 20.	Total.
1 to 2.....				1	1	1			3
2 to 3.....		2	8	14	7	1			32
3 to 4.....			2	10	12	7	3	1	35
4 to 5.....					1	4	3	1	9
Total.....		2	10	25	21	13	6	2	79
BEARDED VARIETIES ²									
1 to 2.....									0
2 to 3.....	4	2	6	6					18
3 to 4.....		16	15	18	6		3		58
4 to 5.....			1	7	4	11	4		27
5 to 6.....			1				5		6
Total.....	4	18	23	31	10	11	12		109

¹ $r = 0.541 \pm 0.045$.² $r = 0.598 \pm 0.041$.

CORRELATION BETWEEN THE PERCENTAGE OF STERILE SPIKELETS AND OTHER CHARACTERS OF THE WHEAT PLANT

For the purpose of studying the relationship between the percentage of sterile spikelets per plant and other characters, 300 plants of the variety Velvet Chaff were pulled, dried, and later carefully measured. The plants had been grown by the centgener method, 6 inches apart each way. The percentage of sterile spikelets was used rather than the actual number, for the reason that the length of spikes, which determines the number of spikelets, varies so greatly. The measurements of length were taken in centimeters and those of weight in milligrams. Biometrical data were secured for the statistical relationship between the percentage of sterile spikelets per plant and (1) the number of culms per plant; (2) the yield of grain per plant; (3) the yield of grain per spike; (4) the length of the culm; (5) the length of the spike; (6) the average weight of the kernel; and (7) the number of spikelets. In the above determinations the plant was used as a unit, the value for each character being determined by taking the average of the respective readings.

CORRELATION BETWEEN THE PERCENTAGE OF STERILE SPIKELETS PER PLANT AND THE NUMBER OF CULMS PER PLANT³

An inspection of Table VI shows only a slight degree of correlation between the percentage of sterile spikelets and the number of culms, which is negative. The coefficient of correlation is -0.076 ± 0.039 . Evidently there exists no appreciable relationship between the percentage of

sterile spikelets and the number of tillers per plant. The less vigorous plants, indicated by the smaller number of tillers per plant, do not show a higher percentage of sterile spikelets than the more thrifty plants.

TABLE VI.—Correlation between the percentage of sterile spikelets per plant and the number of tillers per plant in wheat¹

Percentage of sterile spikelets per plant.	Number of tillers per plant.													Total.
	1	2	3	4	5	6	7	8	9	10	11	12	13	
0 to 3.....	1	2	...	1	...	1	...	1	6
3 to 7.....	2	2	13	4	7	5	8	2	1	1	1	46
7 to 11.....	2	3	4	15	19	16	14	7	2	...	1	1	1	85
11 to 15.....	...	10	14	20	11	11	14	6	3	4	1	1	1	95
15 to 19.....	1	2	7	10	11	4	5	2	1	1	2	1	1	48
19 to 23.....	2	1	4	1	2	1	11
23 to 27.....	...	1	4	2	7
27 to 31.....	...	1	1
31 to 35.....	1	1
Total.....	8	22	46	53	50	38	42	18	7	6	5	3	2	300

$$^1 r = -0.0756 \pm 0.0187.$$

CORRELATION BETWEEN THE PERCENTAGE OF STERILE SPIKELETS AND THE YIELD OF GRAIN PER PLANT

Between the percentage of sterile spikelets and the yield of grain per plant (Table VII) the coefficient of correlation is negative, -0.306 ± 0.035 . This correlation is fairly high and though expressed negatively indicates that the higher yielding plants have a smaller percentage of sterile spikelets than those of low yield.

TABLE VII.—Correlation between the percentage of sterile spikelets per plant and the yield of grain per plant in wheat¹

Percentage of sterile spikelets per plant.		Yield of grain per plant (in milligrams).												Total.	
		0 to 500	500 to 1,000	1,000 to 1,500	1,500 to 2,000	2,000 to 2,500	2,500 to 3,000	3,000 to 3,500	3,500 to 4,000	4,000 to 4,500	4,500 to 5,000	5,000 to 5,500	5,500 to 6,000		6,000 to 6,500
0 to 3.....	1	2	2									1			6
3 to 7.....	1	5	7	6	6	7	2	5	2	3	2	1	1		46
7 to 11.....	2	12	15	23	10	10	4								85
11 to 15.....	6	10	20	16	9	13	5	3	4	1	1	1	1	1	95
15 to 19.....	7	8	12	4	4	7	2			1	2				48
19 to 23.....	5	3	1	1	1										11
23 to 27.....	2	5													7
27 to 31.....	1														1
31 to 35.....	1														1
Total.....	24	42	54	44	43	37	19	12	7	8	3	3	3	3	300

$$^1 r = -0.3057 \pm 0.0353.$$

CORRELATION BETWEEN THE PERCENTAGE OF STERILE SPIKELETS AND THE
AVERAGE YIELD OF GRAIN PER SPIKE

The average yield of grain per spike (Table VIII) was determined by dividing the total weight of grain per plant by the number of spikes per plant. The coefficient of correlation between this yield and the percentage of sterile spikelets is again negative, -0.589 ± 0.025 , which indicates a much closer relationship between the low percentage of sterile spikelets and yield of grain per spike than is shown between the same character and the yield per plant. There is a rather high correlation existing between the percentage of sterile spikelets and the yield of grain per spike.

TABLE VIII.—*Correlation between the percentage of sterile spikelets per plant and the average yield of grain per spike in wheat*¹

Percentage of sterile spikelets per plant.	Yield of grain per spike (in milligrams).																Total.
	50 to 100	100 to 150	150 to 200	200 to 250	250 to 300	300 to 350	350 to 400	400 to 450	450 to 500	500 to 550	550 to 600	600 to 650	650 to 700	700 to 750	750 to 800	800 to 850	
0 to 3.....			1	1	2	..	7	1	2	..	1	6
3 to 7.....			..	1	1	0	7	17	12	14	7	6	1	1	6	..	46
7 to 11.....			6	..	13	13	17	16	13	8	3	2	85
11 to 15.....		2	5	2	11	8	8	9	2	1	1	4	2	1	95
15 to 19.....	2	2	1	1	11	1	1	1	1	48
19 to 23.....	2	1	1	1	2	3	1	11
23 to 27.....			2	4	7
27 to 31.....		1	1
31 to 35.....		1	1
Total.....	4	8	12	33	36	36	46	34	32	26	16	6	2	7	1	1	300

$$^1 r = -0.588 \pm 0.025.$$

CORRELATION BETWEEN THE PERCENTAGE OF STERILE SPIKELETS PER
PLANT AND THE AVERAGE LENGTH OF CULM PER PLANT

In this case the average length of culm per plant (see Table IX) was found by taking the sum of the lengths of the culms of a plant in centimeters and dividing it by the number of culms. The correlation coefficient is -0.448 ± 0.031 . This is a rather high degree of correlation and is expressed as negative, although with reference to the relation of the two characters compared it means that the longer culms tend to form a lower percentage of sterile spikelets. This is what might be expected, since the yield of grain per spike is generally closely associated with the length of spike, and that in turn with the length of culm.

TABLE IX.—Correlation between the percentage of sterile spikelets per plant and the average length of culm in wheat¹

Percentage of sterile spikelets per plant.	Length of culm (in centimeters).												Total.
	65 to 65	65 to 70	70 to 75	75 to 80	80 to 85	85 to 90	90 to 95	95 to 100	100 to 105	105 to 110	110 to 115	115 to 120	
0 to 3.....						1	2	3					6
3 to 7.....					5	6	7	9	12	5	1	1	46
7 to 11.....				3	11	13	17	15	19	6	1		85
11 to 15.....		1	2	6	16	21	21	15	9	3	1		95
15 to 19.....		2		7	13	9	8	7					48
19 to 23.....	1		1	1	4	2	2						11
23 to 27.....		1	1	1	1	3							7
27 to 31.....													1
31 to 35.....				1									1
Total.....	1	5	4	19	50	55	57	49	40	16	3	1	300

$$r = -0.448 \pm 0.0310.$$

CORRELATION BETWEEN THE PERCENTAGE OF STERILE SPIKELETS AND THE LENGTH OF SPIKE PER PLANT

The average length of spike per plant was determined by dividing the sum of the lengths of the spikes per plant by the number of spikes. The calculations were expressed in centimeters. The coefficient of correlation between these two characters is -0.451 ± 0.031 (see Table X). Since the longest spikes usually occupy the longest culms, we should expect the same relationship between the length of spike and percentage of sterile spikelets as was found between the latter character and the length of culm (see Table IX). There is a very close relation, the coefficient of correlation with the culm being 0.448 ± 0.031 , a difference of 0.003 between the two coefficients.

TABLE X.—Correlation between the percentage of sterile spikelets per plant and the average length of spike in wheat¹

Percentage of sterile spikelets per plant.	Length of spike (in centimeters).															Total.
	5.4 to 5.8	5.8 to 6.2	6.2 to 6.6	6.6 to 7	7 to 7.4	7.4 to 7.8	7.8 to 8.2	8.2 to 8.6	8.6 to 9	9 to 9.4	9.4 to 9.8	9.8 to 10.2	10.2 to 10.6	10.6 to 11	11 to 11.4	
0 to 3.....							1	1				2	2			6
3 to 7.....							1	3	5	5	12	8	6	4		46
7 to 11.....						3	4	6	10	14	17	14	5	2		85
11 to 15.....	1	1	2	3	11	11	12	17	15	10	4	3	1	2	1	95
15 to 19.....		1		2	3	12	8	4	8	5	2	1		1		48
19 to 23.....			1	1		2	4	1		1			1			11
23 to 27.....			1	1	3	1										7
27 to 31.....						1										1
31 to 35.....			1													1
Total.....	1	1	3	5	8	21	32	31	46	43	45	28	17	10	3	300

$$r = 0.451 \pm 0.0310.$$

CORRELATION BETWEEN THE PERCENTAGE OF STERILE SPIKELETS AND THE AVERAGE WEIGHT OF KERNEL

To get the average weight of kernel per plant the total weight of kernels per plant was divided by the number of kernels and the result expressed in milligrams. The coefficient of correlation is -0.421 ± 0.032 (see Table XI). This indicates a decided tendency for the heavier kernels to be associated with a low percentage of sterile spikelets. This is in accord with the relations found to exist between the length of culm and spike and the percentage of sterile spikelets. The more vigorous plants, as indicated by an increased length of culm and spike, generally bear kernels of a larger size. Hence, the correlation between the percentage of sterile spikelets and the weight of kernel—in other words, the quality of the grain—is in the same direction and approximates the other coefficients very closely.

TABLE XI.—Correlation between the percentage of sterile spikelets per plant and the average weight of the kernel in wheat¹

Percentage of sterile spike- lets per plant.	Weight of kernel (in milligrams).														Total.
	2 to 4	4 to 6	6 to 8	8 to 10	10 to 12	12 to 14	14 to 16	16 to 18	18 to 20	20 to 22	22 to 24	24 to 26			
0 to 3.....							1	3	1		1			6	
3 to 7.....				2	2	8	4	11	7	8	2	2		46	
7 to 11.....				1	6	17	15	26	13	4	1	2		85	
11 to 15.....		1	3	6	13	11	29	13	13	5		1		95	
15 to 19.....	1	1	1	6	7	15	10	5	1	1				48	
19 to 23.....			3	1	3	1	1	2						11	
23 to 27.....					2	3	2							7	
27 to 31.....			1											1	
31 to 35.....				1										1	
Total.....	1	2	8	17	33	55	62	60	35	18	4	5		300	

$$^1 r = -0.4279 \pm 0.0320.$$

CORRELATION BETWEEN THE PERCENTAGE OF STERILE SPIKELETS PER PLANT AND THE AVERAGE NUMBER OF SPIKELETS PER SPIKE PER PLANT

A relationship is shown below between the percentage of sterile spikelets per plant and the total number of spikelets per plant. The coefficient of correlation is low, -0.152 ± 0.037 (see Table XII). There is only a slight tendency for plants with a low percentage of sterile spikelets to be associated with a large number of spikelets per plant. As the number of spikelets determines to a large extent the length of the spike, it would be supposed that a greater correlation would exist between the number of spikelets and the percentage of sterile spikelets. This may be explained by the fact that the total number of spikelets includes both fertile and sterile spikelets. Also, there may be more or less variation in the condensation of the spikelets which go to make up the spike.

TABLE XII.—Correlation between the percentage of sterile spikelets per plant and the average number of spikelets per spike per plant in wheat¹

Percentage of sterile spikelets per plant.	Number of spikelets per spike.														Total.
	12	13	14	15	16	17	18	19	20	21	22	23	24		
0 to 3.....				1		1		3		1				6	
3 to 7.....				1	7	5	14	10	9					46	
7 to 11.....			2	4	7	4	26	27	15					85	
11 to 15.....			1	5	20	20	20	18	7	1	3			95	
15 to 19.....	1		1	1	4	12	10	12	6					48	
19 to 23.....		1		3		3	1	1	1					11	
23 to 27.....		1	1		1	2	2							7	
27 to 31.....						1								1	
31 to 35.....						1								1	
Total.....	1	2	5	15	39	49	73	71	38	3	3	0	1	300	

$$^1 r = -0.1544 \pm 0.0375.$$

TABLE XIII.—Variation constants in wheat

Plant as the unit.	Mean.	Standard deviation.	Coefficient of variation.
Sterile spikelets....per cent..	11.73 ± 0.198	5.105 ± 0.141	43.51 ± 1.466
Number of tillers per plant....	5.193 ± 0.091	2.336 ± 0.064	44.98 ± 1.468
Yield of grain per plant.mgm..	2,048.333 ± 51.125	1,312.821 ± 36.157	64.09 ± 2.381
Yield of grain per spike.mgm..	379.833 ± 5.475	140.509 ± 3.872	37.02 ± 1.151
Length of culm.....cm..	91.417 ± .366	9.411 ± .259	10.29 ± .286
Length of spike.....cm..	9.019 ± .043	1.113 ± .031	12.34 ± .344
Weight of kernel.....mgm..	15.033 ± .151	3.881 ± .107	25.82 ± .756
Number of spikelets per spike..	17.857 ± .065	1.688 ± .046	9.335 ± .257

Characters.	Coefficient of correlation.
Sterile spikelets and number of tillers per plant.....	-0.075 ± 0.038
Sterile spikelets and yield of grain per plant.....mgm..	- .366 ± .035
Sterile spikelets and average yield of grain per spike.....mgm..	- .589 ± .024
Sterile spikelets and average length of culm per plant.....cm..	- .448 ± .031
Sterile spikelets and average length of spike.....cm..	- .451 ± .031
Sterile spikelets and weight of kernel per plant.....mgm..	- .421 ± .032
Sterile spikelets and the average number of spikelets per spike.....per plant.....	- .152 ± .037

SUMMARY

(1) The number of sterile spikelets per spike in wheat is directly affected by the rate of seeding or the spacing of the plants. The more space allowed each plant the smaller the number of sterile spikelets on each spike.

(2) The bearded varieties of wheat as a class have a higher percentage of sterile spikelets than the beardless varieties. Of the 188 varieties

examined the smallest number of sterile spikelets was found on a beardless variety and the largest number on a bearded variety.

(3) Early seeding seems to increase the percentage of sterile spikelets on each spike. Wheat seeded very late had the smallest percentage of sterile spikelets.

(4) The application of nitrogen alone as a fertilizer produced the lowest percentage of sterile spikelets. Phosphoric acid singly gave the highest percentage of sterile spikelets, while potash was intermediate as to the percentage of sterile spikelets. Where two elements of fertilizers were combined, phosphoric acid and potash gave the highest percentage of sterile spikelets, with nitrogen and phosphoric acid next and nitrogen and potash last. In every instance the check or untreated plots gave a lower percentage of sterile spikelets than those treated with a complete fertilizer.

(5) There is a distinct correlation between the length of spike as expressed by the number of spikelets and the number of sterile spikelets. As the number of spikelets per spike increases (in other words, the length of spike), the number of sterile spikelets becomes greater. That is, varieties with the shorter spikes tend toward a smaller number of sterile spikelets than the varieties with the longer spikes. However, the percentage of sterile spikelets per spike may be greater among the varieties with the shorter spikes, as was shown to be the case where spikes of varying lengths within a single variety were examined.

(6) There is only a very slight correlation between the percentage of sterile spikelets and the number of tillers to each plant.

(7) The yield of grain per plant is correlated to a fair degree with a low percentage of sterile spikelets.

(8) The weight of the kernel or quality of grain is correlated to a considerable degree with a low percentage of sterile spikelets.

(9) The yield of grain per spike, the length of spike, and the length of culm are strongly correlated with a low percentage of sterile spikelets.

(10) There is a slight correlation between the average number of spikelets per spike and a low percentage of sterile spikelets.

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PLATE XXVII .

Comparison of the number of sterile spikelets on bearded and beardless varieties of wheat:

On the left two heads of a bearded variety of wheat showing a large number of sterile spikelets. On the right two heads of a beardless variety showing comparatively few sterile spikelets. Both varieties were grown the same year under like conditions of soil and treatment.

